Exercise 273

For the following exercises, use properties of logarithms to write the expressions as a sum, difference, and/or product of logarithms.

$$\log_5 \sqrt{125xy^3}$$

Solution

There are three properties of logarithms to know.

$$\log(ab) = \log a + \log b \tag{1}$$

$$\log \left(\frac{a}{b}\right) = \log a - \log b \tag{2}$$

$$\log a^b = b \log a \tag{3}$$

Use properties (1) and (3).

$$\log_5 \sqrt{125xy^3} = \log_5(125xy^3)^{1/2}$$

$$= \frac{1}{2} \log_5 125xy^3$$

$$= \frac{1}{2}(\log_5 125 + \log_5 xy^3)$$

$$= \frac{1}{2}(\log_5 125 + \log_5 x + \log_5 y^3)$$

$$= \frac{1}{2}(\log_5 125 + \log_5 x + 3\log_5 y)$$

$$= \frac{1}{2}(3 + \log_5 x + 3\log_5 y)$$

Note that $\log_5 125 = 3$ because $5^3 = 125$.